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Besides the usual “talks” on various topics of interest to the readership of the Séminaires, this volume contains a presentation by A. Lejay of the theory of rough paths, and some of the talks presented in September 2002 at the Journées de Probabilités held in La Rochelle. Other talks given at these Journées will be published in the next volume (volume XXXVIII).

The organizers of the Journées and the Séminaire are thankful to the Conseil Régional de Poitou-Charentes for its support.

The typographical presentation of the contributions has become much more homogeneous than in previous volumes; this meets a demand of our publisher Springer-Verlag, but does not mean that we aim to become a Journal! On the contrary, we are determined to keep using non-scientific acceptance criteria in addition to mathematical ones; the spirit of the Séminaire remains the same: discussing recent results by young (and not so young) researchers, making specialized courses widely available.

This new presentation would not have been possible but for the invaluable TEXnical assistance of Anthony Phan. Many thanks to him for his help and the time and skill he devoted to the finish of this volume.

J. Azéma, M. Émery, M. Ledoux, M. Yor

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Paul André Meyer est décédé subitement à Strasbourg le 30 janvier 2003.

C'est avec une profonde tristesse et une immense reconnaissance que nous saluons la mémoire de celui qui créa et développa sans relâche le Séminaire de Probabilités, et dont les qualités scientifiques et humaines faisaient l'admiration de tous.

Ce volume est dédié à son souvenir.

J. Azéma, C. Dellacherie, M. Émery,
M. Ledoux, M. Weil, M. Yor

An Impression of P. A. Meyer As Deus Ex Machina

Frank B. Knight

When I came to the University of Illinois in 1963 I remember hearing that Meyer had recently spent a semester there, working with J. L. Doob. I believe my first sight of Meyer in person was in 1967 in Madison, Wisconsin at the Chover Symposium on Probability and Potential. Meyer gave a 3-in-1 talk, starting a separate topic on each of the three available blackboards, and ending (no doubt) with a grand unification (but by that time I was lost).

I got the feeling of a sort of Napoleon of probability, both in appearance and in accomplishment. Thus I was agreeably surprised to find at the Dinges and Snell conference in Oberwolfach (1970) that he was in fact quite accessible. There was Meyer standing at the end of a long polished table occupied by 10 or 15 other participants, answering questions in such a calm and resourceful way that I could understand the discussion. It was not long before I, too, was asking questions, and, to make a long story short, we developed a fruitful correspondence. In 1974 I paid my first visit to Strasbourg as a guest of CNRS. That was the year when the volume X of the Séminaire de Probabilités was being aired (and the year of J. L. Doob's 65$^{\text{th}}$ birthday), so this is ancient history and we may skip forward to my second visit to Strasbourg in 1982.

I shall recount the incidents preceding publication of the papers [1] and [2], both because even now they seem novel, and also because they are typical of Meyer's skill and generosity in dealing with colleagues. But the main reason for presenting them now is because they complete the proof of an assertion (Theorem 1.4 of [2]) which Meyer himself found "really beautiful" and which may still be relevant to the subject.

At that time my paper [1] had been submitted to the Ann. Sci. Éc. Norm. Sup., and it turned out that Meyer was acting as both editor and referee. I do not know whether he first obtained the paper as the referee or directly from me. In any case he returned it to me for revision, but with special praise for Theorem 1.4. Meanwhile he obtained a very closely related result which was more general insofar as it included the non-Gaussian processes, but not comparable inasmuch as it applied only for $t = \infty$, whereas Theorem 1.4 was for $t < \infty$. Unfortunately neither of us remarked on this distinction until after Meyer made the decision (sufficiently magnanimous to me) to publish my revision followed by a "Remark" by him containing his simplified proof. And that is how it turned out, except for a breakdown at the last moment. Namely, in the proofreading stage, I discovered a flaw in my original proof of Theorem 1.4. Thereupon (there was no other alternative) I deleted my proof entirely and referred instead to [2]. It was after November 23, 1982 and too late to make any more changes, so it was a shock when I finally noticed the gap between $t < \infty$ and $t = \infty$. Not being able to fill it in despite considerable effort, I again called upon Meyer. In a matter of weeks (or days), he came up with the following simple and brilliant solution. (See also [3] Theorem 3.38.)

Dear Frank

I am sorry to answer you with much delay. Here is the answer to your question. I recall the notation. Given $(\mathcal{F}_t)$ and some nice adapted process (X_t), we set

$$Y_t^\lambda = E\left[\int_t^\infty X_s e^{-\lambda s} ds \,\middle|\, \mathcal{F}_t\right] \qquad Z_t^\lambda = \text{martingale part of } Y_t^\lambda.$$

It is proved that if (Z_t^λ) is known on $[0,\infty[$ for all large λ then one can reconstruct (X_t). Your question is

assume (Z_t^λ) is known on $[0,T]$ for some fixed T.

Can one reconstruct (X_t) on $[0,T]$?

The answer is yes, and the proof is simple, though it took me a long time to find it! Set

$$\tilde{\mathcal{F}}_t = \mathcal{F}_{t\wedge T} \qquad \tilde{X}_t = X_t \text{ for } t \le T, \; E[X_t|\mathcal{F}_T], \; t \ge T$$

Then $\tilde{Y}_t^\lambda = E\left[\int_t^\infty e^{-\lambda s}\tilde{X}_s ds \,\middle|\, \tilde{\mathcal{F}}_t\right] = Y_t^\lambda$ for $t \le T$

and therefore the martingale part is $\tilde{Z}_t^\lambda = Z_t^\lambda$ for $t \le T$,

and since $\tilde{\mathcal{F}}_t = \tilde{\mathcal{F}}_T$ for $t \ge T$ $\tilde{Z}_t^\lambda = Z_T^\lambda$ for $t \ge T$

Therefore if we know (Z_t^λ) on $[0,T]$ we know $(\tilde{Z}_t^\lambda)$ everywhere, so by the above result we know $(\tilde{X}_t)$ everywhere, and hence (X_t) on $[0,T]$.

With kindest regards

André Meyer

References

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3. Knight (F.B.). Foundations of the Prediction Process. Oxford Studies in Probability 1. Clarendon Press, 1982.

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